ON A MODEL FOR DYNAMIC WORK-HARDENING OF A CONTINUOUS MEDIUM IN THE MISES-REUSS THEORY OF PLASTICITY

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The hardness of a metal is characterized by its ability to withstand plastic deformation when some object is pressed into it. In particular, Brinell hardness B is defined by the diameter of the impression d formed by a sphere of diameter D when it is pressed into the metal by an external load of given magnitude P

$$B = \frac{2P}{\pi D (D - \sqrt{D^2 - d^2})} \tag{0.1}$$

The magnitude of B increases nearly proportionately with the ordinate of the static stress-strain curve $(P - \varepsilon)$ at which a metal deforms with work-hardening [1,2]. This effect may be explained in terms of the one-dimensional $(P - \varepsilon)$ diagram as follows. In a metal which has been work-hardened by the attainment of a certain plastic strain ε_p followed by unloading, it is necessary in order to obtain a second plastic strain of the same amount ε_p, to apply a second stress larger than the first stress. It may be said that for a similar deformation there occurs cold working or an increase in hardness, which is also sometimes termed "work-hardening", and which, strictly speaking, is related to the increase in the ordinate of the $(P - \varepsilon)$ curve in the elastic range.

Of technical interest is the problem of increasing the hardness of a metal without having it deform plastically as noted above. Such a problem arises, for example, when it is necessary to increase the hardness of a surface layer of a fabricated part without significantly changing the dimensions or shape of the part. For some time past, this problem has been solved by means of shock waves generated by the explosion of distributed charges on the surface of the fabricated part.

The possibility of producing cold working with insignificant residual deformations may be explained by the fact that the yield limit of many metals increases with an increase in the velocity of deformation. If the stress level attained by an increase of deformation velocity is set, after unloading, by that which appears as a new static yield limit of the material upon repeated loading, then it is obvious that the metal will experience dynamic work-hardening. This is expressed, as in the static case, by an irreversible increase of the ordinate of the $p(\varepsilon)$ curve in a one-dimensional test. However, in contrast to the static case, dynamic work-hardening is determined by the velocity of deformation and the temperature, and is not related to the increase in plastic deformation. If we assume that the cold working is proportional to the dynamic work-hardening, then we obtain a satisfactory explanation of the phenomenon of the increase of hardness of a metal under shock loading.

The dependence of the mechanical behavior of a metal on the velocity of deformation has been investigated in a number of papers, reviews of which are contained in [3, 4]. In addition, we point out [5] from earlier work. Here, in experiments on tin wires for the relationship between the stress p and deformation (strain) velocity, $\dot{\varepsilon} = d\varepsilon/dt$, at the same magnitude of plastic deformation ε_p, the following formula was established

$$p = p_0 + p_1 \ln(\dot{\varepsilon} / \dot{\varepsilon}_0) \qquad (p_0,\ p_1,\ \dot{\varepsilon}_0 = \text{const}) \tag{0.2}$$

In [6], the dependence between p and $\dot{\varepsilon}$ that was obtained for steel noticeably deviated from (0.2).

1. We shall assume that the medium can experience dynamic work-hardening and that this property is determined by certain parameters χ_i. The parameters χ_i must therefore describe the yield state of the medium, so that a variation in the parameters can characterize the acquisition of new properties by the metal in the process being investigated [7].

We assume that the yield surface of the medium is given by differential equations of the form

$$d\sigma_2' \equiv 2\sigma_{ij}'d\sigma_{ij}' = \left\{A(\chi)L[\sigma_2' - H_0(\chi, T)] + \frac{\partial H_0}{\partial \chi}\right\}d\chi + \frac{\partial H_0}{\partial T}dT \tag{1.1}$$

$$\sigma_2' = \sigma_{ij}'\sigma_{ij}', \qquad \sigma_{ij}' = \sigma_{ij} - \tfrac{1}{3}\sigma\delta_{ij}, \qquad \sigma = \sigma_{kk}, \qquad \sigma_{ij} = \frac{P_{ij}}{\rho}$$

Here δ_{ij} is the unit tensor, P_{ij} are the components of the stress tensor in Cartesian coordinates, ρ is the density of the medium, H_0 is a certain function, known from experiments, of the absolute temperature

T and the parameter

$$\chi = \dot{\varepsilon}_{ij}'\dot{\varepsilon}_{ij}', \qquad \varepsilon_{ij}' = \varepsilon_{ij} - \tfrac{1}{3}\varepsilon\delta_{ij}, \qquad \varepsilon = \varepsilon_{kk} \tag{1.2}$$

that characterize the strain velocity, and ε_{ij} are components of the strain tensor. In equations (1.1), $A(\chi)$ and $L(u)$ are certain functions, with $L(u)$ having the following form in the neighborhood of $u = 0$

$$L(u) = u^{\frac{\gamma-1}{\gamma}} + \ldots, \qquad \gamma > 1, \qquad L(0) = 0 \tag{1.3}$$

We shall use Cartesian coordinates in the sense of Euler. Differentiation of components of tensors with respect to time, referred to particles of the medium, are to be understood as material derivatives.

Hence

$$\dot{\varepsilon}_{ij} = \frac{d\varepsilon_{ij}}{dt} = \left(\frac{\partial \varepsilon_{ij}}{\partial t}\right)_x + v^k \left(\frac{\partial \varepsilon_{ij}}{\partial x^k}\right)_t$$

where v^k are velocity components of a particle.

The Pfaffian form (1.1) of three variables σ_2', χ and T has, as a particular solution, the expression

$$\sigma_2' = H_0(\chi, T) \tag{1.4}$$

This solution is the envelope of a one-parameter family of surfaces

$$\sigma_2' = H(\chi, T, \chi^*) \tag{1.5}$$

where χ^* is the parameter of the family. The function $H(\chi, T, \chi^*)$ in (1.5) satisfies equation (1.1) and has the form

$$H(\chi, T, \chi^*) = H_0(\chi, T) + u(\chi, \chi^*) \tag{1.6}$$

where $u(\chi, \chi^*)$ is found from the equation

$$\int_0^u \frac{dx}{L(x)} = \int_{\chi^*}^{\chi} A(x)\, dx \tag{1.7}$$

We shall assume that in the elastic region the parameter χ may change in a backward direction and that the change in χ does not affect the character of the dependence between σ_{ij}, ε_{ij} and T, but that, in accordance with equation (1.4), it determines the domain of variation of this dependence. When the point that represents the state of stress in the space of variables σ_{ij}, χ, T reaches the yield surface (1.4), further yielding of the material occurs according to equation (1.4) if $d\chi \geqslant 0$, and according to equation (1.6) if $d\chi \leqslant 0$. The transition from the flow law (1.4) to the flow law (1.6) takes place at $d\chi = 0$. The value χ^* is defined as that value $\chi = \chi^*$ for which $d\chi = 0$. Fixing χ^* in equation (1.6) extracts from the family a definite surface on which is found the point representing the state of stress of the material in the last phase of plastic deformation before the material re-enters the elastic region upon unloading. The latter surface from the family (1.6) is subsequently retained as a yield surface of the medium.

If in equation (1.1) $A(\chi)$ and $L(u)$ are chosen in such a manner that the function $u(\chi, \chi^*)$ is everywhere non-negative and monotone increasing in χ^*, then it is clear that in the process described the medium will experience dynamic work-hardening.

Hence, we shall proceed from the fact that in the deviator plane of the tensor σ_{ij} there exist an infinite set of Mises circles, depending on T, χ and χ^*.

The dependence of the yield surface on the temperature T was examined in [8,9].

The dependence of the yield limit on the strain velocity cannot, generally speaking, be set within the framework of a one-parameter dependence. However, the assumption that has been made here is extremely simple and it should be examined first.

We assume that in the elastic region, i.e. when $\sigma_{ij}'\sigma_{ij}' < H_0(\chi, T)$, there occurs the finite dependence

$$\varepsilon_{ij} = c\sigma_{ij}' + \tfrac{1}{3} f(\sigma)\,\delta_{ij} + \alpha\delta_{ij}T \qquad (1.8)$$

where α is the coefficient of volume expansion, c is a constant, and $f(\sigma)$ is a certain function of the mean stress.

From (1.8) it follows that the volume deformation is equal to

$$\varepsilon = f(\sigma) + 3\alpha T \qquad (1.9)$$

To within a factor of $1/\rho$, entering into σ_{ij}, the relations (1.8) transform into the usual formulas of thermoelasticity for

$$c = \frac{1}{E}\rho_0(1+\nu), \qquad f(\sigma) = \frac{1}{E}\rho_0(1-2\nu)\,\sigma$$

where E is Young's modulus, ν is Poisson's ratio, and ρ_0 is the initial value of the density of the material.

The variation of the volume deformation of steel with hydrostatic pressure was examined by Bridgeman, and also by Pack, Evans, and James [10,11]. This variation is always elastic and up to pressures of the order of 280,000 atm is well approximated by a quadratic polynomial of the form

$$\frac{\Delta V}{V} = Ap - Bp^2 \qquad (A,\ B = \text{const})$$

where ΔV is the change in volume, and p is the pressure. Hence, up to moderately high pressures, the function $f(\sigma)$ may be represented in the form

$$f(\sigma) = f_0'\sigma + \tfrac{1}{2} f_0''\sigma^2 + \dots \tag{1.10}$$

We note that the calculation of the nonlinear terms in σ in equation (1.8), as compared to σ_{ij}' in the linear approximation, may be justified by the fact that the values of the stress deviator σ_{ij}' are always bounded by the values σ_{ij}' that correspond to the yield surface, whereas the values $\sigma = P_{kk}/\rho$ can be very large for values P_{11}, P_{22} and P_{33} that differ insignificantly.

In the plastic region, i.e. when $\sigma_{ij}'\sigma_{ij}' = H(\chi,\ T,\ \chi^*)$, the strain increment is equal to the sum of the elastic part, $d\varepsilon_{ij}^e$, and the plastic part, $d\varepsilon_{ij}^p$. From (1.8), the elastic increment is equal to

$$d\varepsilon_{ij}^e = c\, d\sigma_{ij}' + \tfrac{1}{3}\delta_{ij} f'(\sigma)\, d\sigma + \alpha\delta_{ij}\, dT \tag{1.11}$$

As in the Reuss theory [1], we take the elastic strain increment in the form

$$d\varepsilon_{ij}^p = \sigma_{ij}' e(\dot\lambda)\, e(\sigma_2' - H)\, d\lambda, \qquad e(u) = \begin{cases} 0, & u < 0 \\ 1, & u \geqslant 0 \end{cases} \tag{1.12}$$

where $\lambda(t,\ x^1,\ x^2,\ x^3)$ is a new unknown function.

Hence, for $\sigma_{ij}'\sigma_{ij}' = H(\chi,\ T,\ \chi^*)$ (or $H_0(\chi,\ T)$)

$$d\varepsilon_{ij} = c\, d\sigma_{ij}' + \tfrac{1}{3}\ \delta_{ij} f(\sigma)\, d\sigma + \alpha\delta_{ij}\, dT + \sigma_{ij}' e(\dot\lambda)\, d\lambda \tag{1.13}$$

In addition to equation (1.13), the complete system in terms of the unknown functions ρ, v^k, σ_{ij}, T and λ for describing the motion of the medium will moreover include:

Continuity equation

$$\frac{d}{dt}\left(\ln\frac{\rho}{\rho_0} + \varepsilon_{kk}\right) = 0, \qquad \dot\varepsilon_{kk} = \frac{\partial v^k}{\partial x^k} \tag{1.14}$$

Momentum equation

$$\frac{dv^i}{dt} = \sigma^{ik}\frac{\partial}{\partial x^k}\left(\ln\frac{\rho}{\rho_0}\right) + \frac{\partial\sigma^{ik}}{\partial x^k} \tag{1.15}$$

Heat flux equation

$$\frac{dQ}{dt} = \frac{dU}{dt} - \sigma_{ij}\frac{d\varepsilon_{ij}}{dt}, \qquad \dot{\varepsilon}_{ij} = \frac{1}{2}\left(\frac{dv^i}{\partial x^j} + \frac{\partial v^j}{\partial x^i}\right) \tag{1.16}$$

where dQ is the external heat acquired by the particles of the medium, and U is the internal energy of the medium per unit mass. Also we have the flow law (1.1), where

$$\chi = \dot{\varepsilon}_{ij}'\dot{\varepsilon}_{ij}' = c^2\dot{\sigma}_{ij}'\dot{\sigma}_{ij}' + e(\dot{\lambda})\,e(\sigma_2' - H)(2c\sigma_{ij}'\dot{\sigma}_{ij}' + \sigma_{ij}'\sigma_{ij}'\dot{\lambda}^2) \tag{1.17}$$

2. We obtain now the density of the internal energy U that enters into equation (1.16) as a function of the determining parameters.

As a consequence of relation (1.8), we have in the elastic range $U = U(\sigma_{ij}, T)$. We apply the second law of thermodynamics to the reversible process of elastic deformation of the particles of the medium. Here, instead of U, it is convenient to consider the density of the free energy $F(\sigma_{ij}, T) = U - ST$, where S is the entropy density. Then

$$\frac{\partial F}{\partial\sigma_{ij}}d\sigma_{ij} + \frac{\partial F}{\partial T}dT - \sigma_{ij}\,d\varepsilon_{ij} + S\,dT = 0$$

Replacing $d\varepsilon_{ij}$ in this equation by expression (1.11) and setting to zero the coefficients of the independent incremental determining parameters, we obtain

$$\frac{\partial F}{\partial\sigma_{ij}} = c\sigma_{ij}' + \frac{1}{3}\delta_{ij}\sigma f'(\sigma), \qquad S = -\frac{\partial F}{\partial T} + \alpha\sigma \tag{2.1}$$

Equations (2.1) are satisfied by the function

$$F = \frac{c}{2}\sigma_{ij}'\sigma_{ij}' + \frac{1}{3}\int\sigma f'(\sigma)\,d\sigma + \Psi(T) \tag{2.2}$$

where $\psi(T)$ is an arbitrary function. From (2.1)

$$S = \alpha\sigma - \psi'(T) \tag{2.3}$$

and therefore

$$U = \frac{c}{2}\sigma_{ij}'\sigma_{ij}' + \frac{1}{3}\int\sigma f'(\sigma)\,d\sigma + \alpha\sigma T + \Phi(T), \quad \Phi(T) = \Psi(T) - T\Psi'(T) \tag{2.4}$$

In the plastic region it is necessary, generally speaking [12], to take $F = F(\varepsilon_{ij}, \sigma_{ij}, T, \chi, \lambda)$. The second law of thermodynamics applied to an irreversible process is expressed by the equation

$$T\,dS = dQ + dQ' \tag{2.5}$$

where $dQ' > 0$ is the uncompensated heat. We shall assume that in the medium considered the uncompensated heat is proportional to the work of the stress forces in plastic deformation, i.e.

$$dQ' = k\sigma_{ij}\,d\varepsilon_{ij}^{p} = k\sigma_{ij}\sigma_{ij}'\,d\lambda = kH(\chi, T, \chi^*)\,e(\dot{\lambda})\,d\lambda \tag{2.6}$$

Here, k denotes a certain function of the determining parameters; and $H(\chi, T, \chi^*)$ denotes the function $H_0(\chi, T)$ when $d\chi \geqslant 0$, and the function $H_0(\chi, T) + u(\chi, \chi^*)$ when $d\chi \leqslant 0$. Then, on the basis of equations (1.16) and (2.5), one may write the equation

$$\frac{\partial F}{\partial \varepsilon_{ij}}\,d\varepsilon_{ij} + \frac{\partial F}{\partial \sigma_{ij}}\,d\sigma_{ij} + \frac{\partial F}{\partial T}\,dT + \frac{\partial F}{\partial \chi}\,d\chi + \frac{\partial F}{\partial \lambda}\,d\lambda - \sigma_{ij}\,d\varepsilon_{ij} + $$
$$+ k\sigma_{ij}'\sigma_{ij}'\,d\lambda + S\,dT = 0$$

Eliminating the dependent differentials of the variable parameters $d\varepsilon_{ij}$ with the aid of equations (1.13), and, in agreement with equation (1.1)

$$d\chi = \left(2\sigma_{ij}'\,d\sigma_{ij} - \frac{\partial H_0}{\partial T}\,dT\right) \Big/ \left\{A(\chi)\,L\,[\sigma_2' - H_0(\chi, T)] + \frac{\partial H_0}{\partial \chi}\right\} \tag{2.7}$$

and further, setting to zero the coefficients of the differentials, we obtain

$$c\,\frac{\partial F}{\partial \varepsilon_{ij}} + \frac{\delta_{ij}}{3}\,[f'(\sigma) - c]\,\frac{\partial F}{\partial \varepsilon_{kk}} + \frac{\partial F}{\partial \sigma_{ij}} + \frac{2\sigma_{ij}'}{A(\chi)\,L\,(\sigma_2' - H_0) + \partial H_0/\partial \chi}\,\frac{\partial F}{\partial \chi} =$$
$$= c\sigma_{ij}' + \frac{\delta_{ij}}{3}\,\sigma f'(\sigma), \qquad \sigma_{ij}'\,\frac{\partial F}{\partial \varepsilon_{ij}} + \frac{\partial F}{\partial \lambda} = (1-k)\,H(\chi, T, \chi^*) \tag{2.8}$$

$$S = \alpha\sigma - \frac{\partial F}{\partial T} + \frac{\partial F}{\partial \chi}\,\frac{(\partial H/\partial T)}{(\partial H/\partial \chi)} - \alpha\,\frac{\partial F}{\partial \varepsilon_{kk}} \tag{2.9}$$

The solution of equations (2.8) and (2.9), which continuously passes over into (2.3), may be put in the form

$$F = \frac{c}{2}\,H(\chi, T, \chi^*) + \frac{1}{3}\int \sigma f'(\sigma)\,d\sigma + \Psi(T) + \varphi(\lambda, T) - \varphi(\lambda_0, T) \tag{2.10}$$

$$H(\chi, T, \chi^*) = \begin{cases} H_0(\chi, T) & \text{for } d\chi \geqslant 0 \\ H_0(\chi, T) + u(\chi, \chi^*) & \text{for } d\chi \leqslant 0 \end{cases} \tag{2.11}$$

Here $\varphi(\lambda, T)$ is a certain function, related to k by the equation

$$k = -\frac{1}{H(\chi, T, \chi^*)} - \frac{\partial \varphi(\lambda, T)}{\partial \lambda} \tag{2.12}$$

The function $u(\chi, \chi^*)$ satisfies the equation

$$\frac{\partial u}{\partial \chi} = A(\chi) L(u)$$

while λ_0 is the value of λ at the instant of transition from the elastic to plastic regions, whereupon $\lambda \geqslant \lambda_0$. The uncompensated temperature is

$$dQ' = \left[H(\chi, T, \chi^*) - \frac{\partial \varphi(\lambda, T)}{\partial \lambda}\right] d\lambda$$

As a consequence of (2.10), the following expression is obtained for the entropy

$$S = \alpha\sigma - \psi'(T) - \frac{\partial}{\partial T}[\varphi(\lambda, T) - \varphi(\lambda_0, T)] \tag{2.13}$$

Hence for $\sigma_{ij}'\sigma_{ij}' = H(\chi, T, \chi^*)$ we obtain (compare [13])

$$U = \frac{cH(\chi, T, \chi^*)}{2} + \frac{1}{3}\int \sigma f'(\sigma)\, d\sigma + \Phi(T) + \sigma\alpha T - T\frac{\partial}{\partial T}[\varphi(\lambda, T) - \varphi(\lambda_0, T)] \tag{2.14}$$

3. To determine the arbitrary function $\Phi(T)$ in (2.5) and (2.14), we examine experiments for the determination of the dynamic compressibility induced by shock waves. Let a shock wave be propagated along an unstressed, quiescent medium of density ρ_0 and absolute temperature $T_0 = 0$. Since the properties of a metal under high pressure approach those of a fluid, immediately in back of the shock wave the stress tensor may be assumed to be spherical, i.e.

$$P_{ij} = \frac{\delta_{ij}}{3} P_{kk}$$

The propagation of the loading shock wave takes place extremely rapidly; therefore heat conduction may be neglected. Hence, the usual conditions on the shock wave [14] lead to the relations

$$\vartheta_n = D\left(1 - \frac{\rho_0}{\rho}\right), \quad \sigma = -3D^2\frac{\rho_0}{\rho}\left(1 - \frac{\rho_0}{\rho}\right), \quad U - U_0 = -\frac{\sigma}{6}\frac{\rho}{\rho_0}\left(1 - \frac{\rho_0}{\rho}\right) \tag{3.1}$$

where ϑ_n is the component normal to the shock wave surface of the particle velocity after passage of the shock wave, D is the velocity of the shock wave front and σ, ρ, U have the previous meanings and refer to magnitudes after passage of the discontinuity. It has been experimentally established that [15]

$$D = a + b\vartheta_n \qquad (a,\ b = \text{const}) \tag{3.2}$$

Hence from (3.1) we find

$$\sigma = -\frac{3a^2 (1 - \rho_0 / \rho)(\rho_0 / \rho)}{[1 - b(1 - \rho_0 / \rho)]^2} \tag{3.3}$$

If the deviator stresses are neglected in comparison with the mean pressure, and the deformation is assumed to be dilatational, then, according to (2.5), $U - U_0$ will have the form

$$U - U_0 = \frac{1}{3}\int \sigma f'(\sigma)\, d\sigma + \sigma\alpha T + \Phi(T) \tag{3.4}$$

From equations (1.14) and (1.9) it follows that

$$\ln \frac{\rho_0}{\rho} = f(\sigma) + 3\alpha T \tag{3.5}$$

Then, comparing (3.1) and (3.4), and taking into account (3.3) and (3.5), we obtain an equation for $\Phi(T)$. If $f(\sigma)$ has the form (1.10), and we restrict ourselves to cubic terms in αT, then from the enumerated equations we find

$$\Phi(T) = \frac{9a^2(\alpha T)^2}{2(3a^2 f_0' - 1)} + \frac{27a^2(3a^4 f_0'' + b + 1)}{(3a^2 f_0'' - 1)^3}(\alpha T)^3 + \ldots \tag{3.6}$$

For a final specification of the medium it is also necessary to determine the functions $H_0(\chi,\ T)$, $u(\chi,\ \chi^*)$ and $\varphi(\lambda,\ T)$ in the expression (2.14).

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